Chapter-1

DIFFERENTIAL EQUATIONS: A differential equation (DE) is an equation containing the derivatives or differentials of one or more dependent variables, with respect to one or more independent variables.

$\frac{dy}{dx} = f(x)$. Here "x" is an independent variable and "y" is a dependent variable.

Examples:

(i) $\frac{dy}{dx} + 2y = 5x$	(iii) $\frac{dy}{dx} + \frac{d^2y}{dx^2} + y = 0$
(ii) $\frac{dy}{dx} + 1 = 0$	(iv) $\frac{\partial z}{\partial x} + \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} = z$

Order and degree of Differential Equation

Differential Equations are classified on the basis of the order. Order of a differential equation is the order of the highest derivative (also known as differential coefficient) present in the equation. If the order of differential equation is 1, then it is called first order. If the order of the equation is 2, then it is called a second-order, and so on.

Examples:

(i) $\frac{dy}{dx} + 2y = 5x$ (order 1 or first order)

(ii) $\frac{dy}{dx} + 1 = 0$ (order 1 or first order)

(iii) $\frac{dy}{dx} + \frac{d^2y}{dx^2} + y = 0$ (order 2 or 2nd order)

Degree of Differential Equation

The degree of differential equation is represented by the power of the highest order derivative in the given differential equation.

(i) $\left(\frac{dy}{dx}\right)^2 + 2y = 5x$ Here, the exponent of the highest order derivative is two and the given differential equation is a polynomial equation in derivatives. Hence, the degree of this equation is 2.

(ii) $\left(\frac{dy}{dx}\right)^3 + 2y = 5x$ Here, the exponent of the highest order derivative is three and the given differential equation is a polynomial equation in derivatives. Hence, the degree of this equation is 3.

(iii) $\left(\frac{d^2y}{dx^2}\right)^2 + \frac{d^4y}{dx^4} + 3\frac{dy}{dx} + y = 9$

Here, the exponent of the highest order derivative is one and the given differential equation is a polynomial equation in derivatives. Hence, the degree of this equation is 1.

(iv) $\left(\frac{d^2y}{dx^2}\right)^4 + \left(\frac{d^4y}{dx^4}\right)^3 + 3\frac{dy}{dx} + y = 9$ Here, the exponent of the highest order derivative is three and the given differential equation is a polynomial equation in derivatives. Hence, the degree of this equation is 3.

Differential Equation Types
Differential equations can be divided into several types namely

- Ordinary Differential Equations
- Partial Differential Equations
- Linear Differential Equations

Ordinary Differential Equations (ODE)
An ordinary differential equation involves function and its derivatives. It contains only one independent variable and one or more of its derivative with respect to the variable.
Examples:

(i) $\frac{dy}{dx} + 2y = 5x$

(ii) $\frac{dy}{dx} + 1 = 0$

(iii) $\frac{dy}{dx} + \frac{d^2y}{dx^2} + y = 0$

Partial Differential Equations (PDE)
An equation involving the partial derivatives of one or more dependent variables of two or more independent variables is called a partial differential equation (PDE).
Examples:

(i) $\frac{\partial z}{\partial x} + \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} = z$

(ii) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 5z$

Linear Differential Equations (LDE)
If the dependent variable (y) and its derivatives are of the first degree, and each coefficient depends only on the independent variable (x), then the differential equation is linear. Otherwise non-linear .
Examples:

(i) $\frac{dy}{dx} + 2y = 5x$ (Linear of order 1)

(ii) $\frac{dy}{dx} + 1 = 0$ (Linear of order 1)

(iii) $\frac{d^2y}{dx^2} + y = 0$ (Linear of order 2)

Applications
Let us see some differential equation applications in real-time.
1) Differential equations describe various exponential growths and decays.
2) They are also used to describe the change in investment return over time.
3) They are used in the field of medical science for modeling cancer growth or the spread of disease in the body.
4) Movement of electricity can also be described with the help of it.
5) They help economists in finding optimum investment strategies.
The various other applications in engineering are: heat conduction analysis, in physics it can be used to understand the motion of waves. The ordinary differential equation can be utilized as an application in engineering field like for finding the relationship between various parts of the bridge.